Learning to Count: An Analysis of the Arithmetic Methods of the Egyptians and the Romans

An Honors Thesis (HONRS 499)

By

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Abstract

The Egyptians and the Romans are known for their great monuments and public works projects. Behind these buildings, however, lies a foundation of mathematics- a foundation that is unknown to the general public. This article is a brief exploration of Roman and Egyptian numerical symbolism and arithmetic methods. It examines each of the four major arithmetic manipulations of numbers (addition, subtraction, multiplication, and division) and looks at the similarities and differences between the two ancient number systems. The paper also hypothesizes the reasons behind the development of these systems as well as other methods that may have been used to perform the manipulations.

Introduction

In the age of computers, smartphones, and calculators it is easy to ignore the basic foundations of mathematics: notation, addition, subtraction, multiplication, and division. If asked to multiply to large numbers, a person today would cringe and reach for the nearest electronic device. However, the ancient Egyptians and Romans performed these operations on a daily basis using not only different numeral systems, but also different methods of performing the operations. They were not aided in their calculations by computer programs or electronic calculators- they were forced to create systems that could be performed by hand (or in some cases with the aid of an abacus).

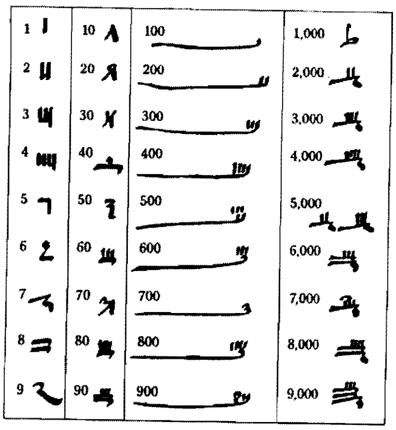
Although considered 'outdated' by the general public, the mathematical systems created by the Egyptians and the Romans were efficient and effective for their time. Because of the ingenuity of both systems, it is important that they not be forgotten. This paper will examine the Egyptian mathematical system and the Roman mathematical system. It will give an overview of the basics of notation for each civilization, as well as explain their methods of addition, subtraction, multiplication, and division. The two systems will be analyzed and compared for their similarities and possible connections. Through the analysis and summary of these two ancient mathematical systems, methods and properties about our own mathematical system can be learned and expanded.

The Egyptians

The Egyptian Counting System

The Egyptian system of counting is a series of representational symbols based on powers of 10. For the numbers 1 through 9, vertical tic marks were used (Gillings 5). When ten hash marks were collected, they were replaced with the symbol that represented a number 10 times larger (Rudman 69). The Egyptians used two different writing styles- hieroglyphic and hieratic (Rudman 70). Hieroglyphs were used for inscriptions on buildings, whereas the hieratic was used for writing on papyrus (Rudman 70). The Egyptians also wrote in the opposite direction from modern day writing- they wrote from right to left (Gillings 5). (It should be noted that in order to avoid any confusion I am keeping the convention of writing from left to right. Thus any Egyptian number the reader encounters has been flipped so as to be able to be read from left to right.) The following table outlines the various symbols used by the Egyptians and their modern day equivalents (Gillings 5):

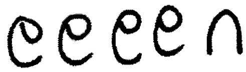
Value	**1**	**10**	**100**	**1000**	**10,000**	**1,000,000**
Hieroglyphic						



The table to the left demonstrates the various hieratic script notations (Irfah, "The Universal History" 170).

Here are a few examples of numbers written in both hieroglyphics and hieratic:

Example 1:

Number	Hieroglyphic	Hieratic
3		
23		
410		
3526		

Addition and Subtraction

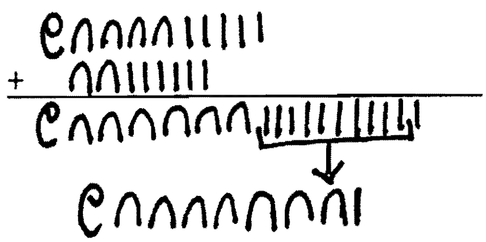
Some historians ignore addition and subtraction- they assume such operations were easily dealt with by the scribes (Gillings 11). However, there are several theories on how the Egyptians performed such operations. One theory suggests the use of addition and subtraction tables, but no tables have been found (Gillings 11). Even if such tables existed, they would be rather large and cumbersome to deal with. Another theory suggests that the Egyptians did the work on a scratch pad using hieroglyphics and then recorded the answer on the papyrus (Rudman 72). The operation would be performed in hieroglyphics because of the ease of addition and subtraction. For addition the scribe only needed to combine the two sets of symbols and replace any set of 10 with the higher valued symbol (see example below). For subtraction, the scribe only needed to “cancel” like symbols and, if needed, convert a larger symbol into 10 smaller symbols (Rudman 70). However, this method seems complicated because of the conversion between scripts, and it assumes the Egyptians would use resources for such a purpose. Below is an example of addition and subtraction using hieroglyphics.

Example 2:

145

<u>+ 26</u>

171



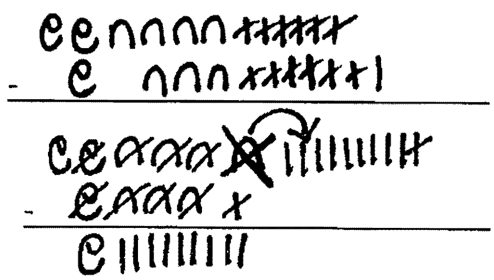
Convert any groups of 10 into a larger symbol

246

<u>-137</u>

109

Cross out any common symbols. Notice how we have to convert ∩ into ||||||||| in order to complete the problem.



The third theory (the most likely in this writer's opinion) is that the Egyptians used an abacus-like device to perform these calculations, and only the answer is written down (Rudman 72). The abacus theory avoids the problem of converting between hieroglyphic and hieratic scripts. Since the abacus does not require the use of symbols, it does not matter which script the scribe uses- after performing the operation the scribe can write down the answer in either script.

Multiplication

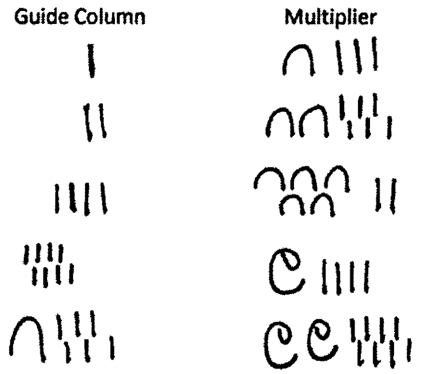
Egyptians did not perform multiplication as we do today. They used a system of doubling and adding (Gillings 18). In a multiplication problem, the scribe chose one of the numbers and repeatedly doubled it, then added the "multipliers" until they summed to the other number (Gillings 18). This is best demonstrated by an example:

Example 3:

Multiply 13x17.

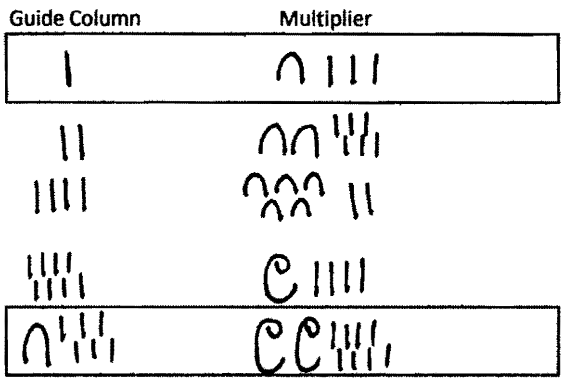
To perform this operation, chose either of the two numbers and write this number across from 1. This will be the multiplier.

Guide Column	Multiplier	
1	13	Now double the values in both columns
2	26	Double again
4	52	Double again
8	104	Double again
16	208	Double again



Now from the guide column, choose the numbers that add up to 17 (in this case the row containing 1 and the row containing 16)

Guide Column	Multiplier
1	13
2	26
4	52
8	104
16	208



Example continued on next page.

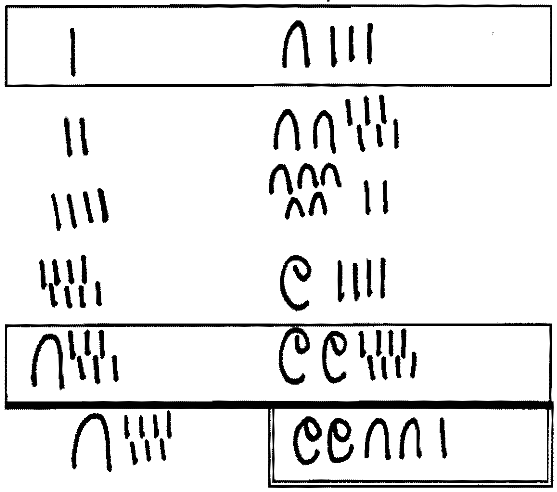
Sum the values from the selected rows column-wise. The answer will be the sum of the multiplier column

Guide Column	Multiplier
1	13
2	26
4	52
8	104
16	208
17	221

Answer!

Now Sum the Squared Rows (Column Wise)

Guide Column Multiplier



The Answer!

It should be noticed that the Egyptian method of multiplication relies on the fact that any whole number can be written by adding some of the terms of the sequence 1, 2, 4, 8, 16, 32, 64, 128, 256… (Gillings 19). This sequence is based on doubling the last term to find the next term. It cannot be determined whether the Egyptians were aware of this fact, but they nevertheless exploited it (Gillings 19).

Division

Division for the Egyptians is similar to multiplication, expect the role of the columns is switched (Rudman 137). [As a reminder or the reader: Dividend/Divisor=Quotient]. Thus the divisor is doubled until reaching a number greater than the dividend, and then the appropriate terms are summed to the dividend. The answer is found by adding the same terms of the "counting" column (Rudman 137). Here is an example to illustrate the procedure:

Example 4:

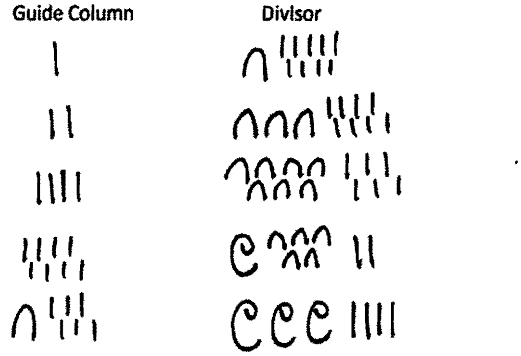
Perform $323 \div 19$

As stated above, this example is the same as multiplication. However, the "doubling number" is now the divisor (in this case 19). The exact same steps are performed as from multiplication.

(Example continued on next page).

Guide Column	Divisor	
1	19	Now double the values in both columns
2	38	Double again
4	76	Double again
8	152	Double again
16	304	Double again

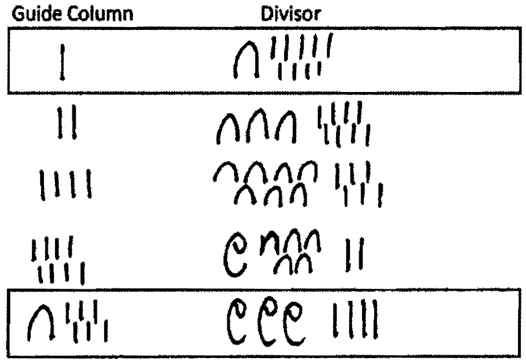
Guide Column Divisor



Now from the divisor column, choose the numbers that add up to the dividend 323 (in this case the row containing 19 and the row containing 304)

Guide Column	Divisor
$\boxed{1}$	$\boxed{19}$
2	38
4	76
8	152
$\boxed{16}$	$\boxed{304}$

Guide Column Divisor



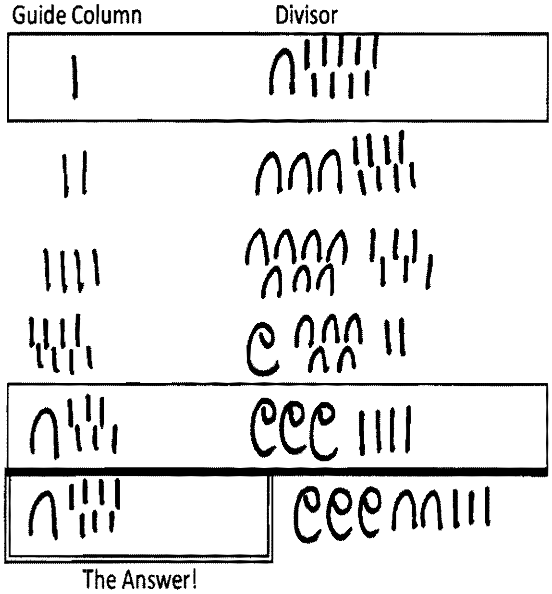
Example continued on next page.

Sum the values from the selected rows column-wise. The answer will be the sum of the guide column

Guide Column	Divisor
1	19
2	38
4	76
8	152
16	304
17	323

Answer!

Now Sum the Squared Rows (Column Wise)



The Romans

The Roman System of Counting

The Roman system of counting uses a set of representational symbols based on powers of ten along with several extra symbols for the numbers 5, 50, and 500 (Ifrah, "From One to Zero" 131). The following is a chart of the traditional Roman numerals (Ifrah, "From One to Zero" 131):

Value	1	5	10	50	100	500
Numeral	I	V	X	L	C	D

Value	1000	5000	10,000	50,000	100,000
Numeral	M or (|)	|))	((|))	|)))	(((|)))

For numbers 1 through 4, the writer would simply collect the appropriate number of I's needed (Ifrah, "From One to Zero" 131). When the writer reached the number 5, he or she would replace the five I's with a V (Ifrah, "From One to Zero" 131). For the numbers 6 through 9, the writer would take the symbol for five and add on the appropriate number of symbols for one to reach the desired number (Ifrah, "From One to Zero" 131). For example the number 8 would be represented by VIII, or V+III=5+3. The concept is the same for the other intermediate numbers of 50 and 500.

Example 5

135 = CXXXV = 100 + 30 + 5

3559 = MMMDLVIII = 3000 + 500 + 50 + 9

*1494 = MCCCCLXXXXIIII = 1000 + 400 + 90 + 4

The last example (labeled with *) is of great importance. The Romans did <u>not</u> use the "subtractive property" of Roman numerals (Flegg 98). The subtractive property is writing a number based on subtracting one from the higher symbol- like representing 4 as IV (5-1=4) or 900 as CM (1000-100=900). These symbols are from the Middle Ages, and they are not found in classical texts (Flegg 98).

The Romans had a difficult time representing multiples of 10 above 100,000 (Ifrah, "The Universal History" 198). The Romans, in fact, did not have a word for one million- they instead said *decies centena milia*, or "ten hundred-thousand" (Ifrah, "The Universal History"198). However, they did use a horizontal line written above the number to represent multiplication by 1,000 and an open box written above the number to represent multiplication by 100,000 (Ifrah, "The Universal History"198).

Example 6

Number	Roman Equivalent
8235	((|))MMMCCXXXV or $\overline{\text{VIII}}$CCXXXV
50,281	|)))CCLXXXI or $\overline{\text{L}}$CCCLXXXI
156,900	(((|))) |))) |)) MLXXX or $\overline{\text{CLVI}}$LXXX
19,235,222	$\lceil\overline{\text{CLXXXXII}}\rceil\,\overline{\text{XV}}$CCXXII

Addition and Subtraction

There is some debate as to how the Romans actually performed addition and subtraction. There is no evidence to be found in the current collection of Classical texts that describes how a Roman would go about doing computation with Roman numerals (Anderson 145). However, there are several theories about how this could be accomplished. One theory postulates the Romans used a “pencil and paper” method- much like is done today (Turner, J. 65). For addition, the two numbers would be written down and then combined (Turner, J. 65). If ten of the same type of symbol were collected, then they would be replaced with the symbol worth ten times as much. For subtraction, the two numbers would be written down and similar terms would be crossed out and eliminated (Turner, J. 65). This is shown in Example 7. Another theory is the Romans used a calculating device to perform these operations- such as an abacus or a counting table with pebbles (Turner, J. 69).

Example 7:

2723	Write out both numbers	MMDCCXXIII
+ 392		+ CCCLXXXXII
3115	Combine both groups of symbols	MMDCCCCCLXXXXXXIIIII
	Convert any group of 5 symbols into the higher valued symbol	MMDDLLXV
	Simplify	MMMCXV

312	Write out both numbers (larger on top)	CCCXII
- 85		- LXXXV
237		
	Borrow 10 from X and 10 from C	CCLLXVVII
		- LXXXV
	Borrow 5 from L	CCLXXXXXXXVVII
		- LXXXV
	Cross out any common symbols	CC~~LXXX~~XXXV~~V~~II
		- ~~LXXXV~~
	Write down any remaining symbols	CCXXXVII

Multiplication

There is little evidence for how the Romans multiplied two numbers. There is one example of such a table in the works of Victorius of Aquitaine (Maher 382-383). Besides the use of tables W. French Anderson advocates a method of multiplication not unlike the modern method, except with the multiplication occurring from the highest numbers to the lowest numbers (Anderson 146). For example:

Example 8:

123	Start by writing out the two numbers	CXXIII
X 12		x XII
1476		
	Multiply CXXIII by X	MCCXXX
	Multiply CXXIII by I	CXXIII
	Multiply CXXIII by I	+ CXXIII
	Add	MCCCCXXXXXXXIIIIII
	Convert notation as necessary	MCCCCLXXVI

This method was also shown to be viable on an abacus by J. Turner (Turner, J. 71).

Example 9:

527
x 15
7905

First multiply DXXVII by X (MCCLXX) and place this value in the abacus

•		•	
M	C	X	I
	•	•	
	•	•	

Now Multiply DXXVII by V (MMDCXXXV) and add this value to the abacus

•	•		•
M	C	X	I
•	•		
•	•		
	•		
	•		

The answer is then $\overline{V}$MMDXXXV

There is also some discussion that Roman multiplication was a system of doubling and halving (Stern). The two numbers to be multiplied would be written in two columns. The first column would be halved (ignoring the remainder if the number was odd) and the second column would be doubled (Stern). When the first column was halved down to one, the rows where the first column is even would be crossed out, and the remaining numbers in the second column would be summed (Stern). This is best demonstrated by an example:

Example10:
Multiply 73 by 21.

Halving	Doubling		Halving	Doubling
73	21	The dividend (72) goes into the halving column and the divisor (21) goes into the doubling column. Half 73 (discard remainder) and double 21	LXXIII	XXI
36	42	Half and double again	XXXVI	XXXXII
18	84	Half and double again	XVIII	LXXXIII
9	168	Half and double again	VIIII	CLXVIII
4	336	Half and double again	IIII	CCCXXXVI
2	672	Half and double again	II	DCLXXII
1	1344	Half and double again	I	MCCCXXXXIIII

Now select the rows which have an ODD number in the halving column.

Example continued on next page

Example 10 Continued:

Halving	Doubling
73	21
36	42
18	84
9	168
4	336
2	672
1	1344
	1533

Answer!

Now sum the selected values from the DOUBLING column. This is the solution after simplification

Halving	Doubling
LXXIII	XXI
XXXVI	XXXXII
XVIII	LXXXIII
VIIII	CLXVIII
IIII	CCCXXXVI
II	DCLXXII
I	MCCCXXXXIIII
	MDXXXIII

The Answer!

Division

Just as with the other basic operations, there is little evidence and many theories on how the Romans divided their numbers. One theory suggests the use of continuous subtraction of the dividend by the divisor [Dividend/divisor=quotient] (Turner, L). The divisor is subtracted until the answer is less than the dividend (with the remaining numbers being a remainder), and the quotient is calculated by the number of times subtraction was performed (Turner, L). For larger dividend and smaller divisor (such as dividing 1980/15), the divisor can be multiplied by a factor of 10- a relatively easy exercise involving changing each symbol for the next larger symbol- and the quotient is calculated in the same fashion (Turner, L).

Example 11:

Divide 3013 by 131

Dividend	Count
3013	
- 1310	10
1703	
- 1310	10
393	
- 131	1
262	
- 131	1
131	
- 131	1
0	23

Start by writing out the dividend. Notice that 10x131 is less than the dividend. So start off by subtracting 10x131= 1310. Write down a count of 10

Again subtract 1310 and add ten to the count

Now notice 131 cannot be subtracted again. Subtract 131 and add 1 to count

Now sum the count column. This is the answer!

Dividend	Count
MMMXIII	
- MCCCX	X
MDXXIII	
- MCCCX	X
CCCLXXXXIII	
- CXXXI	I
CCLXII	
- CXXXI	I
CXXXI	
- CXXXI	I
	XXXIII

Answer!

Another theory hypothesizes division similar to the modern method of long division (Anderson 147). The only difference from the modern method is that any multiple of the divisor can be subtracted from the dividend in any order (Anderson 147). This can be demonstrated by an example:

Example 12:

Divide 3751 by 31.

```
   121
31)3751
   3100
    651
    620
     31
     31
      0
```

After writing down the problem, multiply XXXI by C (yields MMMC) and subtract (yields DCLI). Multiply XXXI by XX (Yields DCXX) and subtract. Multiply XXXI by I and subtract.

```
         CXXI
XXXI)MMMDCCLI
     MMMC
         DCLI
         DCXX
             XXXI
             XXXI
```

As with multiplication, the Romans could have also used an abacus like device to divide (Turner, J 73). The method described by J. Turner is essentially the same method of continuous subtraction discussed above with the aid of an abacus (73).

Example 13:

Divide 1679 by 23.

First place the dividend (1679) into the abacus

	•	•	•
M	C	X	I
•	•	•	•
		•	•
			•
			•

Next, subtract 10x73=730 from the dividend. (Yields 949)

	•		•
M	C	X	I
	•	•	•
	•	•	•
	•	•	•
	•	•	•

Again subtract 10x73=730. (Yields 219)

			•
M	C	X	I
	•	•	•
	•		•
			•
			•

Now subtract 73. (Yields 146)

			•
M	C	X	I
	•	•	•
		•	
		•	
		•	

Example continued on next page.

Subtract 73 (Yields 73)

		•	
M	C	X	I
		•	•
		•	•
			•

Subtract 73 (Yields 0)

The total subtraction is
X+X+I+I+I = XXIII (23)

The Comparison and Analysis of Notation and Methods of Operation

The Notation:

Besides for the differences in symbols, the Roman and Egyptian methods of numerical notation are similar. Both systems are a base ten additive system. A base ten additive system is where new symbols are introduced based on powers of ten (1; 10; 100; 1000; etc), and the value of a number is found by summing the values of the symbols. The position of the symbol in an additive system is not tied to its value. For example the number 29 can be represented in Roman numerals as XXVIIII (10+10+5+1+1+1+1) or XIIIIXV (10+1+1+1+1+10+5). Although both represent a possible representation of the number 29, general convention (and logic) usually dictate the order of writing the symbols. Our own system of numerical notation- the Hindu-Arabic system- is a base ten positional system. For each power of ten, a new placeholder is added. The position of a symbol in the number represents its value. For example the numbers 29 and 92 are not the same number. However, the Romans added some intermediate values of notation- namely V (5), L (50) and D (500). The addition of these symbols made it easier and shorter to represent some numbers. Without such symbols 999 would be written as CCCCCCCCCXXXXXXXXXIIIIIIIII instead of DCCCCLXXXXVIIII (a considerable improvement).

An advantage the Egyptian and Roman systems have over the Hindu-Arabic system is the ease of understanding. Both have visual representation of numbers- IIII is a clear representation of four whereas the symbol 4 has no intrinsic meaning. For the common masses of the Romans and Egyptians who were illiterate, these visual symbols would be a way for information to be conveyed easily. These representations can also be better understood in the future (say 10,000 years) because of their visual nature.

While these visual representations have some advantages, there is a disadvantage in writing large numbers with both of the Roman and Egyptian systems. For example writing 999 requires the use of twenty-seven symbols in the Egyptian hieroglyphic system and fifteen symbols in the Roman system. Neither system has an effective and clear way of notating large numbers. This can have particularly disastrous consequences- especially in dealing with money. As the Roman Emperor Galba can attest, the misunderstanding of the numerical system can lead to financial ruin (Universal history 200). Emperor Tiberius had to execute the will of his mother, and she had written in her will to pay Galba the sum of CCCCC, or 50,000,000 sesterces (Ifrah, "The Universal History" 200). However, Tiberius said the will only stipulated a sum of $\overline{\text{CCCCC}}$, or 500,000 sesterces, to be paid to Galba- a difference of 49,500,000 sesterces (Ifrah, "The Universal History"200).

The Egyptian numerical system also differed from the Roman because of the two different scripts. The development of a different notational system indicates the Egyptians recognized the inefficiency of their notation. Though the hieratic system was also an additive base ten system, it was nevertheless much easier to write than the tradition hieroglyphic script. The hieratic script still suffered the same problem as both the Roman and Egyptian hieroglyphic systems: too many symbols were needed to represent large numbers. One of the greatest advantages the Hindu-Arabic number system has over both the Roman system and the Egyptian system is its limited number of symbols. Only ten different symbols are needed to write any possible number (with no gimmicks needed).

Addition and Subtraction

Only one fact is certain about Egyptian and Roman addition and subtraction: no one can say for certain how these operations were performed. There does not seem to be much evidence for how either civilization carried out these operations, however, there are many theories about how the ancients would have performed them.

Regardless of how they performed these operations, both methods lend themselves to addition and subtraction. Addition is especially easy with both symbols- one only needs to combine the two numbers and up-convert any groups of ten (or possibly five for Roman numbers). The ease of summing numbers in both systems is due to the additive property of both numbers. Modern addition is a rather tedious task when compared to the Egyptian and Roman methods. Students now have to learn to first add the one's place, then carry over another number to the next tens column if necessary, then add the tens column (and so on). How much simpler it would be to only group symbols!

Subtraction is a straightforward operation. It is a simple matter of deleting similar symbols. The cumbersome part of subtraction is the need for "borrowing" from a higher symbol. This is an inelegant way of dealing with numbers. With such a visual number system, it seems more likely that a subtraction problem would be approached more like an addition problem (Gillings 13). Instead of saying "What is 17 minus 8?" (which would force you to use borrowing), the question could be worded "What needs to be added to 8 in order to get 17?" (Gillings 13). By stating the problem this way, the "borrowing" issue is eliminated.

Asking a subtraction question in this way would also be easier to perform on an abacus. Instead of having to convert a higher-valued marker into ten lower-valued markers and adding them to the lower column (which is not possible on a typical abacus since most only have ten markers in each column), markers would only need to be added until the desired number was reached. The downside of the method, however, the user must keep track of the numbers subtracted. For example:

Example 10:
What needs to be added to 133 to get 915?

First put into the abacus 134.

C	X	I
•	•	•
	•	•
	•	•
		•

Start with the I column. Add I to this column to get V.

		•
C	X	I
•	•	
	•	
	•	

Now move onto the X column. Add LXXX to this column to yield X

		•
C	X	I
•	•	
•		

Now move onto the C column. Add DCC to yield DCCCC.

•		•
C	X	I
•	•	
•		
•		
•		

Notice how the abacus ends up as DCCCCXV. Summing the values added to the abacus yields DCCLXXVIIII- the solution to the problem.

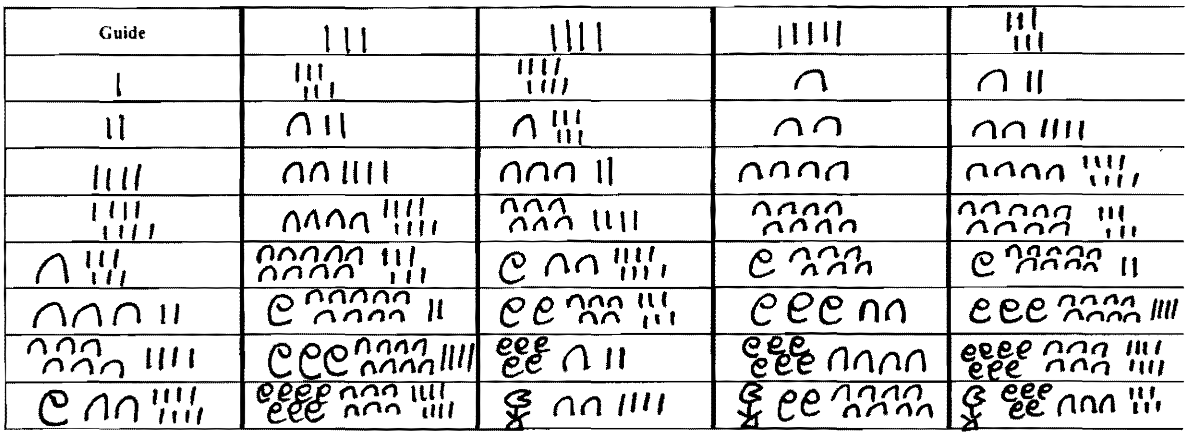
Imagine having to type in the Roman numerals DCCCCLXXXXVIIII for 999. Trying to use this notation on a modern calculator would be an annoying task. However, both the Egyptians and the Romans had access to an abacus (or something similar) for basic mathematical operations. The abacus lends itself to both sets of notation because of the visual nature of the device. It works exactly the same way as the “pencil and paper” method without the need to use the costly resources of a pencil and paper. Of all the possible theories for how these two civilizations performed addition and subtraction, the abacus method is the most plausible. The abacus is easy to operate and not hard to construct. The only issue with the abacus is the inability to check the work (a similar problem faced today with the use of calculators).

Credence should not be given to the chart theory for addition and subtraction. If such a chart were to exist, it would have to be astronomically large in order to cover enough numbers to be of any use. Along with its potentially large size (not considering the large amount of resources it would require to produce such a chart), this theory is extremely inelegant and clunky. Because of both the visual and additive nature of the Egyptian and the Roman systems, such a chart would be almost redundant. Why would the ancients bothered to write down that II+II=IIII or III-I=II in a chart form?

Multiplication:

Thanks in part to several mathematical papyri, scholars are fairly certain on how the Egyptians performed multiplication. The benefit of the Egyptian system is that a person only needs to know how to double numbers and add. Our multiplication system relies on pure memorization of times-tables. While for exceedingly large numbers (say larger than 100,000) the Egyptian system is difficult, it does provide an elegant way around rote memorization of multiplication tables.

That is not to say the Egyptians would not have made use of such a table. In many ways the Egyptian multiplication table would be easier to understand than a modern day equivalent. Below is a sample of such a table:



The Roman method of doubling and halving, on the other hand, is a more complicated method of multiplication and requires the performer to know division by 2 (halving). While this is not a considerable increase in difficulty, it does add to the complexity of the problem. Two different operations would have to be performed for each row (halving and doubling) while the Egyptian method only requires one operation (doubling). The doubling and halving method would also make it more complicated to create multiplication charts because for any number you would need to have both a doubling column and a halving column. Thanks to Victorius of Aquitaine, there is a copy of a multiplication table (Maher 383). A partial reproduction of the table is shown:

Guide	**II**	**III**	**IIII**	**V**
$\overline{\mathrm{I}}$	$\overline{\mathrm{II}}$	$\overline{\mathrm{III}}$	$\overline{\mathrm{IIII}}$	$\overline{\mathrm{V}}$
DCCCC	$\overline{\mathrm{I}}\mathrm{DCCC}$	$\overline{\mathrm{II}}\mathrm{DCC}$	$\overline{\mathrm{III}}\mathrm{DC}$	$\overline{\mathrm{IIII}}\mathrm{D}$
C	CC	CCC	CCCC	D
V	X	XV	XX	XXV

On the surface both Egyptian multiplication and Roman doubling and halving multiplication seem rather similar. Both use addition (for the most part) to achieve multiplication. However from a mathematical standpoint, these two methods could not be further apart. The Egyptian method relies on a simple trick from number theory- namely that any whole number can be represented as a sum of distinct powers of two. For example, $97 = 64 + 32 + 1 = 2^6 + 2^5 + 2^0$. On the other hand, the Roman doubling and halving requires the use of binary numbers to explain (Stern).

Compared to the modern method of multiplication, the Roman abacus method is the most similar. The Roman abacus method requires the knowledge of the one-through-nine multiplication table (not a complicated feat as there are only 81 entries). From a mathematical standpoint, this method is the most efficient. It does not require pencil and paper to compute (See Roman Multiplication), and is capable of handling large numbers with ease. This method also does not require the user to keep track of columns, nor does it require the guessing of which rows need to be added. The abacus multiplication method also has the obvious advantage of being able to be calculated on an abacus.

Division

Because of several mathematical papyri from ancient Egypt, scholars have a fairly comprehensive view on Egyptian division. The Egyptian method of division relies on the doubling of two columns of numbers. However, a careful observer would notice that

Egyptian division is the same as a multiplication problem! Instead of asking themselves "Divide 35 into 5 parts", the Egyptians are asking the following "What number needs to be multiplied by 5 to get 35?" They can even use the same doubling chart.

From a mathematical standpoint, it is rather incredible that the Egyptians realized they could use their multiplication system to perform division. This is something that is lost even in modern day due to the use of long division. Instead of thinking of multiplication and division as two sides of the same coin (like the Egyptians), modern day notation separates these two operations [This author remembers learning multiplication several months before division was even mentioned].

This clever observation by the Egyptians was lost on the Romans. The Romans, much like modern day, performed their division without regard to how they performed multiplication (regardless of which theory of Roman division). In fact the continuous subtraction theory for division is completely different from the Egyptian method. The continuous subtraction theory is a brute-force and clumsy approach to division on paper. While it does yield an answer, it is not a very efficient method of division because of the requirement to keep a tally of the number of times subtraction is performed (especially for large numbers).

The Roman long division method is the most intriguing theory because it is strikingly similar to modern day division. However, trying to perform such a calculation on an abacus is a rather tedious (and rather difficult) feat. The continuous subtraction method, on the other hand, is much easier to perform on an abacus. This method only requires the user to keep in mind the number of times he or she subtracts the divisor from the dividend. From a visual standpoint it is easier to perform because the only multiplication necessary is by 10- this just shifts the columns of the divisor left. From this point of view, the continuous subtraction method is better at performing division than long division (one is not required to know any other multiplies besides powers of 10). Though it may not be as efficient as long division, the continuous subtraction method is nevertheless useful for division.

Conclusion

Though separated by thousands of years, the Egyptians and the Romans had strikingly similar mathematical systems. Both number systems are additive base ten systems, both are easy to use on an abacus (or other counting device), and both performed addition and subtraction the same way. The only glairing differences in the systems are the introduction of V, L, and D into the Roman numerals and the performance of multiplication and division. Despite these differences, the respective numerical and arithmetic systems of both civilizations fulfilled their needs and allowed them to flourish and grow.

If nothing else, the Egyptians and the Romans should be admired for their ingenuity in creating their mathematical systems. They found elegant ways to exploit the properties of their numbers systems to solve problems. Necessity is the mother of invention, and both the Egyptians and the Romans were creative in their invention. It is easy to ignore or forget these ancient methods of mathematics, but there is much that can be learned from them. It only requires us to turn off the calculator, pick up a pencil and paper, and attempt math problems with an open mind.

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